

Worksheet #2 01-11-24 FTC Practice

① $y = f(x) = \begin{cases} 2 & \text{on } 0 < x < 2 \\ x & \text{on } 2 < x < 3 \\ -x & \text{on } 3 < x < 5 \end{cases}$ $A(x) = \int_0^x f(t) dt$

$A(1) = \int_0^1 f(t) dt = 2(1) - 2(0) = 2$ $A'(1) = \text{Derivative of integral} = f(x) = 2$

$A(2) = \int_0^2 f(t) dt = 2(2) - 2(0) = 4$ $A'(2) = 2$

$A(3) = \int_0^3 f(t) dt = \frac{3^2}{2} - \frac{0^2}{2} = 9/2$ $A'(3) = 3$

$A(4) = \int_0^4 f(t) dt = \frac{(4)^2}{2} - \frac{0^2}{2} = 8$ $A'(4) = 2$

The max value of $A(x)$ on interval $[0, 5]$ is 5.

The max value of $A'(x)$ on interval $(0, 5]$ is 3.

② $v(t) = \begin{cases} 2 & \text{on } 0 < x < 1 \\ -x & \text{on } 1 < x < 4 \\ x & \text{on } 4 < x < 7 \end{cases}$

$s(0) = \int 2 = 2(0) = 0$ $v(2) = 1$

$s(2) = \int -x = -\frac{x^2}{2} = -\frac{2^2}{2} - \frac{0^2}{2} = -2$ $v(4) = -1$

$s(4) = \int -x = -\frac{x^2}{2} = -\frac{4^2}{2} - \frac{0^2}{2} = -8$ $v(6) = 1.8$

$s(6) = \int x = \frac{x^2}{2} = \frac{6^2}{2} - \frac{0^2}{2} = 18$

The max value of $s(t)$ on the interval $[0, 7]$ is $s(7) = \frac{7^2}{2} = 24.5$

The max value of $v(t)$ on the interval $[0, 7]$ is $v(7) = 3$

The min value of $v(t)$ on the interval $[0, 7]$ is $v(4) = -1$

③ Net & Total change

A Evaluate $\int_{-2}^2 |x^2 - 4| dx$ and $|\int_{-2}^2 (x^2 - 4) dx|$

$\int_{-2}^2 x^2 - 4 $	$\int_{-2}^2 (x^2 - 4) dx$
$\int_{-2}^2 \frac{x^3}{3} - 4x$	$\int_{-2}^2 \frac{x^3}{3} + 4x$
$F(2) - F(-2)$	$F(2) - F(-2)$
$(\frac{8}{3} - 4(2)) - (-\frac{8}{3} + 8)$	$(-\frac{8}{3} + 4(2)) - (-\frac{8}{3} - 8)$
$= -5\frac{1}{3} - 10\frac{2}{3}$	$= 5\frac{1}{3} + 10\frac{2}{3}$
$= -\frac{16}{3} - \frac{32}{3} = -\frac{48}{3}$	$= \frac{16}{3} + \frac{32}{3}$
$= -\frac{32}{3}$	$= \frac{32}{3}$

$\int_{-2}^2 |x^2 - 4|$ can be either $-32/3$ or $32/3$ but $|\int_{-2}^2 (x^2 - 4) dx|$ is only $32/3$

B Evaluate $\int_{-3}^3 |x^2 - 4| dx$ and $|\int_{-3}^3 (x^2 - 4) dx|$

$\int_{-3}^3 x^2 - 4 $	$\int_{-3}^3 (x^2 - 4) dx$
$\int_{-3}^3 \frac{x^3}{3} - 4x$	$\int_{-3}^3 \frac{x^3}{3} + 4x$
$F(3) - F(-3)$	$F(3) - F(-3)$
$(9 - 12) - (-9 + 12)$	$(-9 + 12) - (-9 - 12)$
$-3 - 21$	$3 + 21$
$= -24$	$= 24$
	$= -3 - 21 $
	$= 24$

$\int_{-3}^3 |x^2 - 4|$ can be either -24 or 24 , but $|\int_{-3}^3 (x^2 - 4) dx|$ is only 24 .

* Net change = the change relative from start to end that has happened, like displacement.
 * Total change = all the change that has happened, like distance

